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FAILURE OF COMPOSITE STRUCTURES IN THE PRESENCE OF CREEP

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Abstract. *The present paper describes some results on the modeling of failure behavior of composite laminates in the presence of large displacements and creep. It follows the research of Marques and Creus (1994), and uses part of its formulation and computer code. We reproduce here the essentials of that formulation and discuss in some detail the procedure used for the analysis of viscoelastic progressive failure of plates and shells. Illustrative examples are included showing the performance of the code.*

Keywords: *Composites, Failure, Viscoelasticity, Finite Elements.*

1. INTRODUCTION

Many composite materials of polymeric base show a viscoelastic behavior that is enhanced by changes of temperature or humidity content (Marques and Creus, 1994). These constitutive characteristics influence the failure behavior, particularly when nonlinear geometrical effects are important, and represent one of the limiting design parameters for advanced composite structures expected to operate for long periods of time, as in civil engineering applications (Barbero, 1998).

The present work continues the research of Marques and Creus (1994), and uses part of its formulation and basic code. For the sake of completeness, we reproduce here the essentials of that paper. Then, the formulation needed for the numerical analysis of viscoelastic progressive failure of composite laminates is given, together with illustrative examples. In Section 2 we resume the geometrically nonlinear formulation, that follows Bathe (1996). In Section 3 we describe the viscoelastic formulation. In Section 4 we discuss the failure criterion used (the Maximum Strain Criterion) and the algorithm for stiffness degradation during progressive failure. Failure criteria are described in many books, as for example in Vinson (1993); a critical discussion is given in Hart-Smith (1993). To be used with linear viscoelasticity, the Maximum Strain Criterion is the most convenient. Degradation criteria are described for example in Lee (1982), Tolson and Zabaras (1991) and Cheung et al. (1995); we adopted this formulation, with small changes.

In Section 5 we give some details of the numerical implementation, including the incremental-iterative strategies. We use an incremental state variable formulation which one is general and efficient. It can be extended to the cases of aging viscoelasticity (Masuero and Creus, 1993) and to non-linear viscoelasticity (Masuero and Creus, 1993) and can be as efficient as Laplace transform methods (Pacheco and Creus, 1997).

We present examples aimed to check the accuracy of the code. In Example 1 we check the nonlinear geometric and viscoelastic deformations algorithms against a closed solution. In example 2 we check the viscoelastic and failure algorithms in bending. In example 3 we analyze failure in a creep-buckling problem, involving the three effects. Finally, we show a more complex example, the progressive failure of a shell under constant load. This example is not compared with a benchmark, but the solution looks reasonable; we are presently looking for well documented experimental results in order to falsify and eventually improve our numerical model.

Most of the examples chosen are simple, in order to allow comparison with closed solutions. Nevertheless, the formulation implemented, that allows the representation of deferred failure, observed in real composite structures, can be extended to more complex real situations. Additional details may be found in (Oliveira, 1999).

2. FINITE ELEMENT MODEL

We follow the general procedure described by Bathe (1996), but including the effects of viscoelastic and hygrothermal deformations. As seen in Marques (1994) this leads to an incremental relation of the form

$$\left(\left[{}_0^k K_L\right]+\left[{}_0^k K_{NL}\right]\right)\{U\}=\left\{{}^{k+1}P\right\}-\left\{{}_0^k F\right\}+\left\{{}_0 F^v\right\}+\left\{{}_0 F^T\right\}+\left\{{}_0 F^H\right\} \tag{1}$$

where $\left[{}_0^k K_L\right]$ and $\left[{}_0^k K_{NL}\right]$ are the linear and non linear tangent stiffness matrices, respectively, corresponding to step k, $\left\{{}^{k+1}P\right\}$ is the vector of external nodal forces at step $k+1$, $\left\{{}_0^k F\right\}$ is the vector of nodal point forces equivalent to the element stresses at the step k and, finally, $\left\{{}_0 F^v\right\},\left\{{}_0 F^T\right\}$ and $\left\{{}_0 F^H\right\}$ are the vectors of viscoelastic, thermal and hygroscopic loads, respectively.

3. VISCOELASTIC MATERIAL MODELING

In the presence of mechanical and hygrothermal loads, the constitutive relations of the layer, referred to the principal material directions, may be written as

$$\varepsilon_i(t)=\int_0^t D_{ij}(T,H,t-\tau)\frac{\partial\sigma_j(\tau)}{\partial\tau}d\tau+\int_{T^*}^{T}\alpha_i(T,H)\,dT+\int_{H^*}^{H}\beta_i(T,H)\,dH \tag{2}$$

where $\varepsilon_i(t)$ are the components of the strain vector $\{\varepsilon\}=\{\varepsilon_{11},\varepsilon_{22},2\varepsilon_{12},2\varepsilon_{13},2\varepsilon_{23}\}$ and $\sigma_j(t)$ are the components of the stress vector $\{\sigma\}=\{\sigma_{11},\sigma_{22},\sigma_{12},\sigma_{13},\sigma_{23}\}$, at time t. The components ε_{33} and σ_{33} are not considered. T and H indicate the temperature and moisture content, respectively.

In Eq. (2), $D_{ij}(T,H,T-\tau)$ are the creep functions corresponding to components ε_i and σ_j, $\alpha_i(T,H)$ are the thermal expansion coefficients and $\beta_i(T,H)$ are the hygroscopic expansion coefficients, that in general depend on moisture and temperature conditions. T^* and H^* are the temperature and moisture values corresponding to the strain-free state. ε_i, σ_j, T

and H are field variables and thus change in general from point to point, even when this dependence is not explicitly stated.

The viscoelastic strain $\bar{\varepsilon}_i$ is formed by two components: one instantaneous ε_i^e and one deferred ε_i^v, given respectively by

$$\begin{aligned} \varepsilon_i^e &= D_{ij}(T_0, H_0, 0)\sigma_j(t) \\ \varepsilon_i^v &= \sum_{p=1}^{M}\sum_{s=1}^{5} q_{is}^p(t) \end{aligned} \tag{3}$$

where q_{is}^p are the state variables and M is the number of significant terms in the series and depends on the accuracy desired. It can be shown that these state variables are given by

$$\frac{\partial q_{is}^p}{\partial t} + \frac{q_{is}^p}{\theta_{is}^p} = \frac{D_{is}^p}{\theta_{is}^p}\sigma_i(t) \tag{4}$$

This is a system of linear first-order uncoupled differential equations that together with the initial condition $q_{is}^p = 0$ at $t = 0$ allows the determination of the state variables knowing the stress history. This system may be solved incrementally by finite differences as indicated in Section 5.

4. FAILURE ANALYSIS

To implement a progressive failure analysis we need criteria that consider the different failure modes. The criteria of Hashin (1980), Lee (1980 and 1982) and the Maximum Strain are well known and have been implemented in the code. Being the goal of this work to analyze viscoelastic progressive failure, we choose a limit criterion in terms of strain, the Maximum Strain Criterion.

In order to realize the above-mentioned analysis we use a degradation model for the layers in the cumulative failure stages, that introduces some material stiffness reductions after the detection of the first ply failure. The analysis ends with the failure of the last ply. The real degradation process is very complex and still not well understood. We choose a simplified degradation model, which eliminates terms in the material stiffness matrix, according to the failure mode. This proceeding has been proposed in some others works (Lee (1982), Tolson and Zabaras (1991) and Cheung et al. (1995)).

4.1 Maximum Strain Criterion

In the Maximum Strain Criterion the failure takes place when one of the following conditions, referred to the principal directions of the laminate (directions *1*, *2*, *3*), are satisfied.

Extension

$$\left(\frac{\varepsilon_{11}}{X_{\varepsilon t}}\right)^2 = 1 \qquad \left(\frac{\varepsilon_{22}}{Y_{\varepsilon t}}\right)^2 = 1 \tag{5}$$

Shortening

$$\left(\frac{\varepsilon_{11}}{X_{\varepsilon c}}\right)^2 = 1 \qquad \left(\frac{\varepsilon_{22}}{Y_{\varepsilon c}}\right)^2 = 1 \tag{6}$$

Distortion

$$\left(\frac{\varepsilon_{12}}{S_{\varepsilon A}}\right)^2 = 1 \qquad \left(\frac{\varepsilon_{13}}{S_{\varepsilon A}}\right)^2 = 1 \qquad \left(\frac{\varepsilon_{23}}{S_{\varepsilon T}}\right)^2 = 1 \tag{7}$$

being $X_{\varepsilon t}$ the extension limit strain in the direction *1*, $X_{\varepsilon c}$ the shortening limit strain in the direction *1*, $Y_{\varepsilon t}$ the extension limit strain in the direction *2*, $Y_{\varepsilon c}$ the shortening limit strain in the direction *2*, $S_{\varepsilon A}$ the distortion limit strain in the plans *1-2* e *1-3* and $S_{\varepsilon T}$ the distortion limit strain in the plan *2-3*. These values have to be determined experimentally.

4.2 Layer Degradation Model

To use the Maximum Strain Criterion in progressive failure analysis we consider three different failure modes: failure in the fiber direction (principal direction *1*), failure in the direction normal to the fiber (principal direction *2*) and shear failure. Extension and shortening failures can occur in both directions *1* and *2*.

In the fiber mode the stiffness matrix becomes

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \tag{8}$$

In the matrix mode the corresponding stiffness matrix takes the form

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{Bmatrix} = \begin{bmatrix} C_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \tag{9}$$

In the shear failure we must consider different situations. If the shear acts on plans *1-2* or *2-3*, we have matrix failure with the reduced stiffness given by Eq. (9). If the shear acts on plane *1-3*, a kind of "delamination" (Oliveira, 1999) occurs in the layer matrix, and the stiffness corresponding to shears ε_{13} and ε_{23} vanish. So, the stiffness matrix is written

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \tag{10}$$

5. NUMERICAL SOLUTION

The numerical solution of the problem formulated in Section 2 is implemented through an incremental-iterative procedure using Eq. (1). For the solution of the non-linear equilibrium equations, we can use the Newton-Raphson Method or the Generalized Displacement Control Method proposed by Yang e Shieh (1990).

In the Newton-Raphson method we have a prescribed load increment. The problem with this method is the numerical instability that occurs when the determinant of the stiffness matrix approaches to zero.

In the Generalized Displacement Control Method the increment is fixed by the algorithm. The load factors for the iterations corresponding to a step *k* are

$$\Delta\lambda_k^1 = \pm\Delta\lambda_1^1 \frac{\langle U_1 \rangle_1^1 \{U_1\}_1^1}{\langle U_1 \rangle_{k-1}^1 \{U_1\}_k^1} \qquad \text{for } i=1 \tag{11}$$

and

$$\Delta\lambda_k^i = -\frac{\langle U_1 \rangle_{k-1}^1 \{U_2\}_k^i}{\langle U_1 \rangle_{k-1}^1 \{U_1\}_k^i} \qquad \text{for } i \geq 2 \tag{12}$$

where $\Delta\lambda_1^1$ is the initial load factor that have to be chosen and $\langle \ \rangle$ indicates line vector.

The generalized displacement control method is specially indicated for post critical analyses. In viscoelastic analyses, when we are interested in the study of the behavior of the structure subjected to a constant load, we cannot use this method. In both methods a displacements convergence criterion is used.

For the determination of the viscoelastic loads in Eq. (1) we must integrate the state variables in time. Adequate algorithms can be seen in (Creus, 1986), (Masuero and Creus, 1993) and (Marques and Creus, 1994).

6. EXAMPLES

6.1 Viscoelastic Buckling Analysis

A panel of homogeneous and isotropic viscoelastic material with dimensions specified in Fig. 1 and an initial deflection in the *z* coordinates given by $z = 0.2(sen\,\pi x / L)$ is subjected to a load in the *x* direction.

Figure 1 – Panel geometry

Because of the symmetry in geometry and loading, only half the panel is to be analyzed. Figure 2 shows the mesh used.

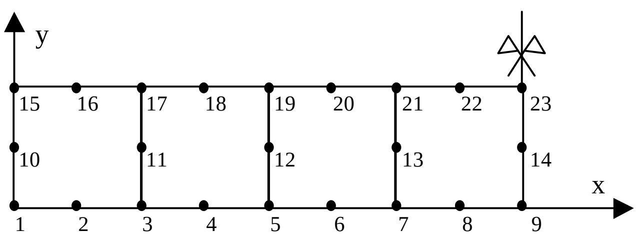

Figure 2 – Mesh, four eight-node elements

The panel is modeled with ten layers of a viscoelastic material with the following properties: E_0 = 132,30 GPa; E_1 = 132,30 GPa; η_1 = 1323,0 GPa; θ = 10 sec.

The analytical values of buckling load P_D and deflection $b(t)$ are given by (Creus, 1986)

$$P_D = \frac{\pi^2 E_\infty I}{L^2}, \text{ where } E_\infty = \frac{E_0 E_1}{E_0 + E_1}$$

$$b(t) = \frac{P\, b_0 (P_D - P_E)}{(P_E - P)(P_D - P)} \; exp\left(\frac{P_D - P}{P - P_E} \; \frac{E_0 + E_1}{\eta} t \right) + \frac{P_D b_0}{P_D - P}$$

Figure 3 shows the comparison between analytical and numerical deflections considering three different loads. We can observe that the results are fairly close.

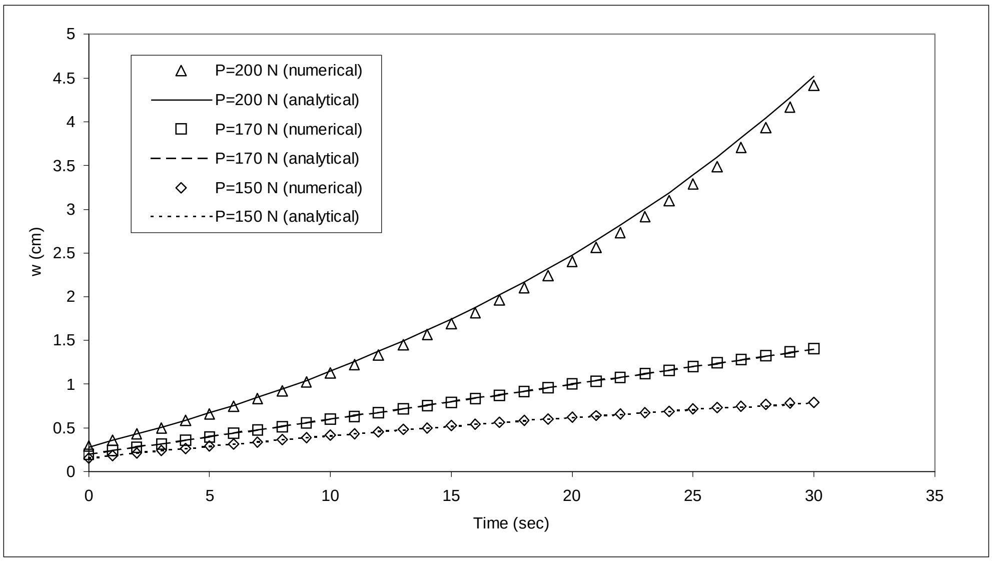

Figure 3 - Comparison between analytical and numerical results

6.2 Viscoelastic failure analysis in bending

We consider a panel with the same dimensions used in the first example (see Fig. 1), but without the initial deflection, analyzed with the same mesh (see Fig. 3). The panel is made of four layers with equal thickness and fiber orientation.

The material of the layers has the following characteristics: E_{11} = 132,30 GPa; E_{22} = 10,75 GPa; G_{12} = G_{13} = 5,65 GPa; G_{23} = 3,40 GPa; ν_{12} = 0,24; θ = 10 s. The panel is simply supported and subjected to a uniform load in the z direction.

We perform a failure analysis using the Maximum Strain Criterion with the following deformation limits

Layers 1 and 4:
$X_{\varepsilon t}$ = 1,1431x10^{-3}; $X_{\varepsilon c}$ = 1,2811x10^{-3}; $Y_{\varepsilon t}$ = 4,0698x10^{-4};
$Y_{\varepsilon c}$ = 4,0698x10^{-4}; $S_{\varepsilon A}$ = 1,5363x10^{-3}; $S_{\varepsilon T}$ = 1,9853x10^{-3}
Layers 2 and 3:
$X_{\varepsilon t}$ = 5,7155x10^{-3}; $X_{\varepsilon c}$ = 6,4055x10^{-3}; $Y_{\varepsilon t}$ = 2,0349x10^{-3};
$Y_{\varepsilon c}$ = 2,0349x10^{-3}; $S_{\varepsilon A}$ = 7,6815x10^{-3}; $S_{\varepsilon T}$ = 9,9265x10^{-3}

The increase of the deflection in time at the center of the panel, for a load of 75 KPa, is shown in Fig. 4.

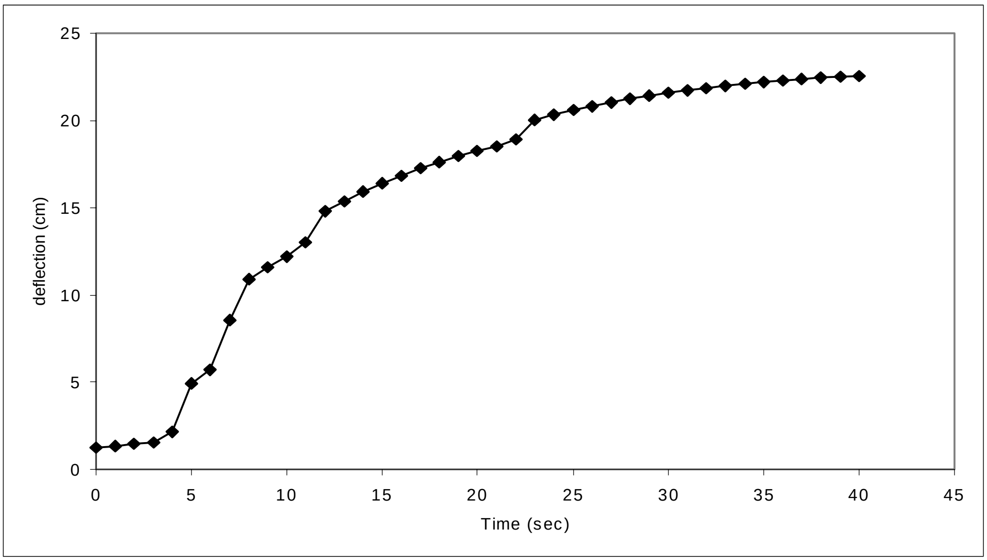

Figure 4– Deflection-time plot for viscoelastic failure analysis in bending

6.3 Viscoelastic failure analysis in buckling

In this example we consider the same panel analyzed in the first example (see Fig. 1). The following limits of deformation were adopted

Layers 1 and 10:
$X_{\varepsilon t}$ = 1,00x10^{-6}; $X_{\varepsilon c}$ = 1,00x10^{-6}; $Y_{\varepsilon t}$ = 4,00x10^{-7};
$Y_{\varepsilon c}$ = 4,00x10^{-7}; $S_{\varepsilon A}$ = 1,50x10^{-6}; $S_{\varepsilon T}$ = 2,00x10^{-6}
Layers 2 and 9:
$X_{\varepsilon t}$ = 2,00x10^{-6}; $X_{\varepsilon c}$ = 2,00x10^{-6}; $Y_{\varepsilon t}$ = 8,00x10^{-7};
$Y_{\varepsilon c}$ = 8,00x10^{-7}; $S_{\varepsilon A}$ = 3,00x10^{-6}; $S_{\varepsilon T}$ = 4,00x10^{-6}
Layers 3 and 8:
$X_{\varepsilon t}$ = 4,00x10^{-6}; $X_{\varepsilon c}$ = 4,00x10^{-6}; $Y_{\varepsilon t}$ = 16,00x10^{-7};
$Y_{\varepsilon c}$ = 16,00x10^{-7}; $S_{\varepsilon A}$ = 6,00x10^{-6}; $S_{\varepsilon T}$ = 8,00x10^{-6}
Layers 4 and 7:
$X_{\varepsilon t}$ = 11,00x10^{-6}; $X_{\varepsilon c}$ = 11,00x10^{-6}; $Y_{\varepsilon t}$ = 30,00x10^{-7};
$Y_{\varepsilon c}$ = 30,00x10^{-7}; $S_{\varepsilon A}$ = 15,00x10^{-6}; $S_{\varepsilon T}$ = 17,00x10^{-6}
Layers 5 and 6:
$X_{\varepsilon t}$ = 16,00x10^{-6}; $X_{\varepsilon c}$ = 16,00x10^{-6}; $Y_{\varepsilon t}$ = 64,00x10^{-7};
$Y_{\varepsilon c}$ = 64,00x10^{-7}; $S_{\varepsilon A}$ = 17,50x10^{-6}; $S_{\varepsilon T}$ = 18.00x10^{-6}

In Fig. 5 we present the variation of the maximum deflection with time. After the failure of some layers, the applied load, that is 20N, reaches the value of the viscoelastic buckling load and the deflections grow rapidly until the final failure.

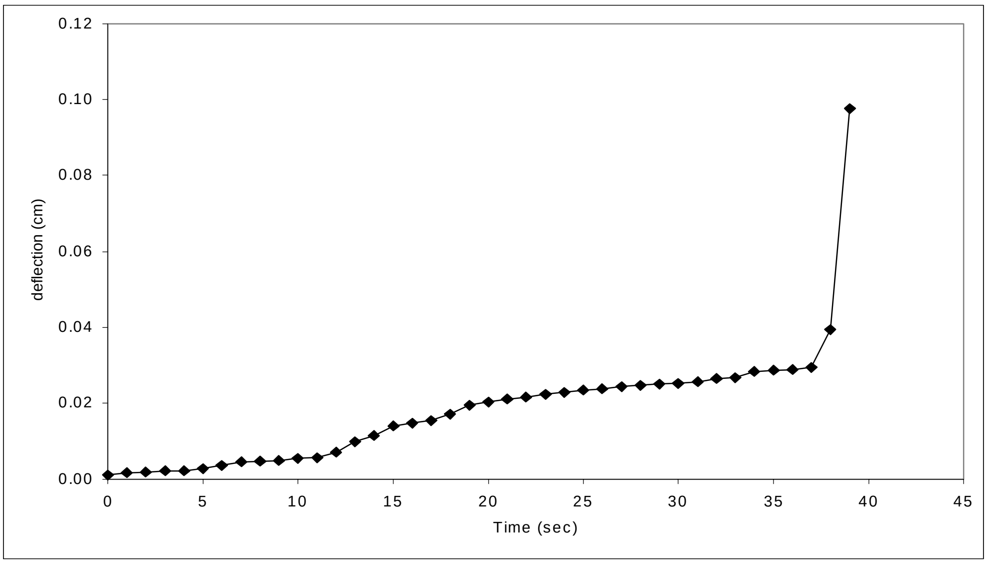

Figure 5 – Deflection-time plot for viscoelastic failure analysis in buckling

6.4 Analysis of a spherical shell

A spherical shell is subjected to a pair of loads as shown in Fig. 6. Due to the symmetry of the problem only one octant of the shell is modeled using a 48 eight-node elements mesh. The Lee’s failure criterion (Lee, 1982) was used in the analysis.

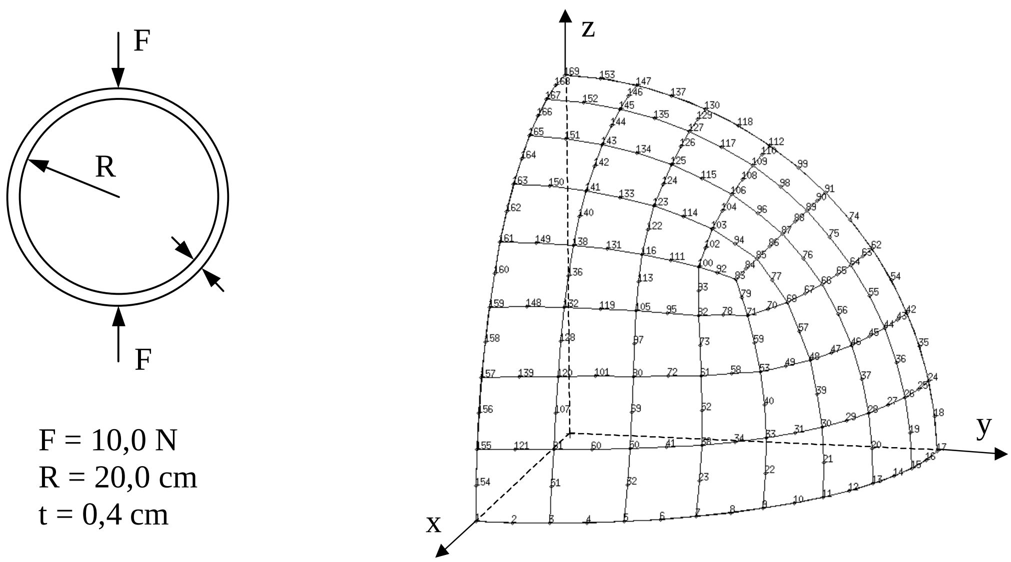

Figure 6 – Load configuration, geometry and used mesh

The shell has 10 layers of carbon-epoxy oriented at $(0/45/90/135/180)_s$, with the following properties: E_1 =180,00 GPa; E_2 = 10,60 GPa; $G_{12} = G_{13} = G_{23}$ = 7,56 GPa; ν_{12}=0,28; X_t = 1500 MPa; X_c = 1500 MPa; Y_t = 40 MPa; Y_c = 250 MPa; S_A = 68 MPa; S_T = 68 MPa

Failure configurations for the 10^{th} layer of the shell are presented together with the failure loads in Fig. 7. Only the localization of the fail region is shown, without indication of failure type. It is also shown the variation of the load during the failure process. In this example, that was run with the Generalized Displacement Control Method we can see the loss of strength of the structure during the progressive failure process.

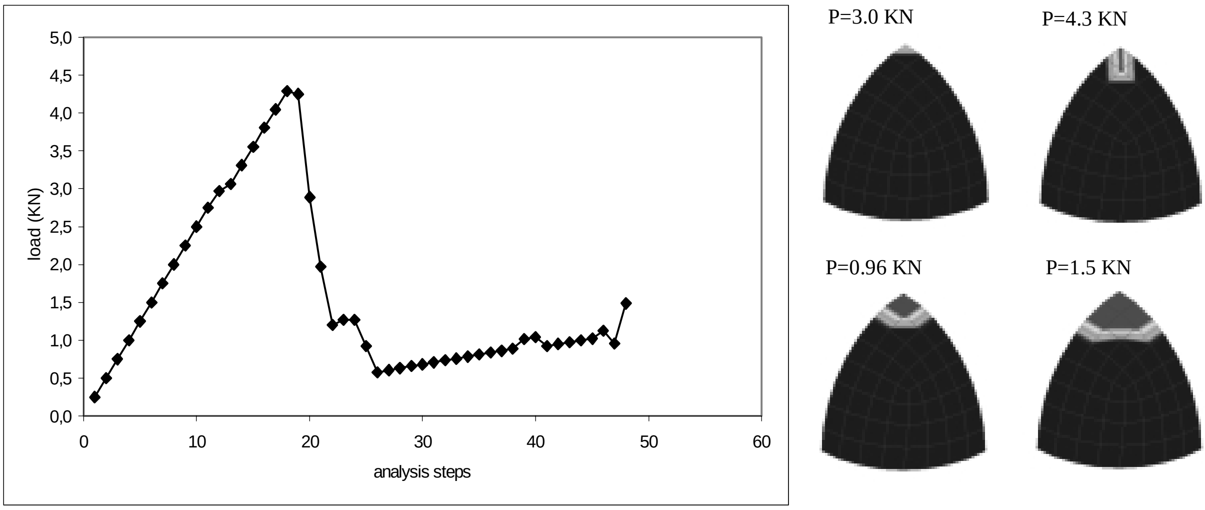

Figure 7 – Load-time variation and failure configurations

7. FINAL REMARKS

Time dependent failure due to viscoelastic behavior is an important problem in composite structures of polymeric matrix. In this paper a numerical procedure for the modeling of this process is described. The examples show good approximation of analytical results and validate the procedures for nonlinear geometric, viscoelastic and progressive failure analyses.

Most of the examples are simple, in order to allow comparison with closed solutions. Nevertheless, the formulation implemented, that allows the representation of deferred failure observed in real composite structures, is quite general and can be extended to more complex real situations.

Acknowledgements

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